LBL-14726 Rev. c.2
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Engineering & Technical Services Division

Submitted to Health Physics

VARIANCE AND REGRESSION ANALYSES OF MOYER MODEL PARAMETER DATA AND THEIR VARIATION WITH PRIMARY PROTON ENERGY

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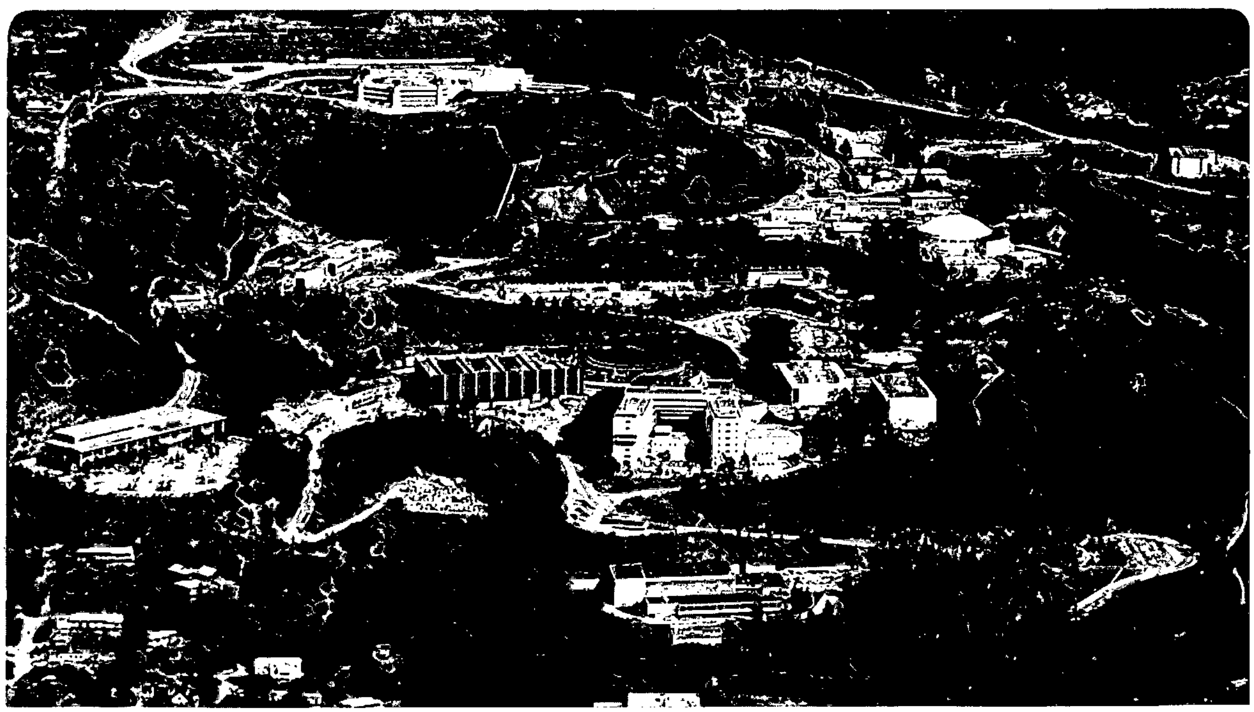
April 1983



Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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LBL-14726 (Revised)
1st April 1983
Submitted to Health
Physics as a Note

VARIANCE AND REGRESSION ANALYSES OF MOYER MODEL PARAMETER DATA AND THEIR VARIATION WITH PRIMARY PROTON ENERGY

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This work was supported by the U.S. Department of Energy under Contract No. DE-AC03-76SF00098; by the European Centre for Nuclear Research, Geneva; by the Institute of High-Energy Physics, Academia Sinica, Beijing, People's Republic of China.

ABSTRACT

Experimental values of the Moyer Model Parameter, H_o, are summarized and presented as a function of proton energy, E_p. The variation of $H_o(E_p)$ with E_p is studied by regression analysis. Regression Analysis of the data under log-log transformation gives the best value for the exponent m of 0.77 ± 0.26, but a t-test did not reject m = 1 (p = 20 percent). Since m = 1 was not excluded, and a Fisher's F-test did not exclude linearity, a linear regression analysis was performed. A line passing through the origin was not rejected (Student's t-test, p = 30 percent) and has the equation: $H_o(E_p) = (1.61 \pm 0.19) \times 10^{-13}$ Sv m^2/GeV. It is suggested that improved data are needed.

VARIANCE AND REGRESSION ANALYSES OF MOYER MODEL PARAMETER DATA AND THEIR VARIATION WITH PRIMARY PROTON ENERGY

"Experience Joined With Common Sense,
to Mortals is a Providence"

"The Spleen"
Matthew Green 1696-1737

INTRODUCTION

Stevenson et al. (St 82) have recently reviewed the available experimental determinations of the Moyer model parameter, H_0 (Mo 62).

H_0 is defined by the equation:

$$H = (1/r^2)\ H_0 \exp(-\beta\theta) \exp(-d/\lambda) \tag{1}$$

where H is the dose equivalent on the shield surface per interacting proton and the symbols r, θ and d are explained in Figure 1. The angular distribution parameter, β, and the attenuation length, λ, are well determined both by theoretical and experimental means (Pa 73, St 82).

The empirically determined values of H_0 are summarized in Table 1.

H_0 is a function of the primary proton energy and it is the statistical analysis of the data of Table 1, both by analysis of variance and by regression analysis, in order to determine the functional form of this variation, that this paper describes.

Table 1. Summary of published values of moyer model parameters $H_0(E_p)$.

Primary Proton Energy, $[E_p]$ (GeV)	Moyer Parameter, $[H_0(E_p)]$ ($Sv\ m^2$)		Source
7.4	1.4	10-12	Sh69, St69
7.4	2.1	10-12	Sh69, St69
10.0	0.96	10-12	Ho 66
13.7	2.5	10-12	Gi 68
13.7	3.1	10-12	Gi 68
21.0	1.6	10-12	Ho 79
23.0	3.5	10-12	Ma 79
25.5	3.3	10-12	Gi 68
25.5	5.0	10-12	Gi 68
25.5	6.6	10-12	Ro69, St82
30.0	3.4	10-12	Aw 70

This note gives details of the regression analysis briefly reported by Stevenson et al. (St 82). It describes the application of Fisher's F-test to the hypothesis that the experimental data are linearly related (both untransformed and in log-log transformation); tests various hypotheses relating the Moyer Parameter, H_o, with primary proton energy E_p and discusses the need for more experimental data. Finally, it is suggested that the techniques of statistical analysis described here may be usefully applied in the future as more experimental data are accumulated.

2. ENERGY VARIATION OF H_o

It is important to understand the variation of H_o with proton energy, both so that the experimental determinations of H_o at various proton energies may be combined to permit accurate interpolation and perhaps, more importantly, to allow extrapolation to higher energies. Such a need arose, for example in the design of shielding for the 50 Gev Beijing Proton Synchrotron (Ch 80, Li 79).

Since the principle use of the Moyer Model is in the calculation of transverse shielding, we are interested in the global production of neutrons at large angles to the interaction target, as determined outside substantial shielding. At energies below 1 Gev there is evidence that the global production of neutrons is roughly proportional to neutron energy (for a summary see Pat 73). If an exponential variation of the form:

$$H_o(E_p) = kE_p^m \qquad (2)$$

is assumed, a value of m = 1 sets an upper limit to the variation of neutron production with proton energy and this is therefore a conservative assumption for extrapolating the experimental determinations of H_o to higher energies.

There has been some speculation in the literature as to the value of the coefficient m. Lindenbaum pointed out that the production of shower particles varied as $E^{0.25}$ and suggested a value of m = 0.50 for fast nucleons, intermediate between that for shower particles and low energy neutrons (Li 61, Pa 73). The data obtained from Monte-Carlo calculations of the Hadron Cascades generated in matter by high-energy protons suggest a value of m = 0.75 (Fe 72).

Until recently there were insufficient experimental data to empirically investigate the relationship between H_o and E_p but the experimental data of Table 1, shown in Figs. 2 and 3, now make this possible.

3. STATISTICAL ANALYSIS OF DATA

The number of data points severely limits the analysis and our purpose here is to first show that the assumption of linearity between the random variables $H_o(E_p)$ and E_p or between the random variables $\log_{10}H_o(E_p)$ and $\log_{10}E_p$ is not excluded by the experimental data.

The sequence of statistical tests described is:

(a) Fisher's F-Test of the hypothesis of linearity of the data and of the data under log-log transformation

(b) Regression analysis under the assumptions

$$H_0(E_p) = a + bE_p$$

and $\log_{10} H_0(E_p) = K + m \log_{10}E_p$

(c) Student's t-Test of log-log transformed data for m = 1

(d) Student's t-Test for a = o

(e) Linear regression analysis with $H_o(E_p) = b'E_p$

(f) Analysis of variance techniques to calculate 95 percent confidence bands to regression lines.

The strategy in using a Fisher's F-test to test the assumption of a linear relationship between two variables is to generate the appropriate F-statistic (Ch 74). A series of assumptions about the data generate a measure of variability which becomes the denominator of the F-ratio (in this case the assumptions are (i) that the values of x or (E_p or $\log_{10}E_p$) are fixed, i.e., not random variables, (ii) the values of y, ($H_o(E_p)$ or $\log_{10}H_o(E_p)$), are independent random variables, (iii) for each value of x the distribution of the y values is normal, (iv) the variance of the dependent variable, y, is the same for all values of the independent variable). A null hypothesis is added (in this case linearity) and incorporated into a measure of variability which is the numerator of the F-ratio. If the null hypothesis is true (i.e., the two measures of variability differ only because of random influences) the F-statistic follows an F-distribution and probabilities are obtained from standard tables (Fi 80, Sn 80).

If the data describe a situation where x and y are linearly related and several y-values have been obtained at a given value of x, a test for linearity may be derived by comparing values of $\overline{y}_i$ (the mean value of y at x_i) and $\hat{y}_i$ (the value of y at x_i estimated from regression analysis).

Under the assumption of linearity $\bar{y}_i$ and $\hat{y}_i$ should have approximately the same values, or $(\bar{y}_i - \hat{y}_i)$ should differ only by random error. The appropriate F-statistic is obtained by partitioning the total variability which comprises the Regression Sum of Squares and the Residual Sum of Squares. In turn the Residual Sum of Squares is made up of the within-group sum of squares and the about-regression sum of squares. It may be shown that the appropriate F-ratio for testing the hypothesis of linearity is:

$$F = \frac{\text{About-Regression Mean Square}}{\text{Within-Group Mean Square}}$$

which has an F-distribution with k-2, N-k degrees of freedom (N = total number of observations, k = number of groups) [Ch 74].

The results of these tests are summarized here but are described in more detail by Lieu et al. (Li 82).

3.1 ANALYSIS OF VARIANCE

Since the data of Table 1 have more than one determination of $H_o(E_p)$ at energies of 7.4, 13.7 and 25.5 Gev the assumption of linearity of the data may be tested by analysis of variance techniques (Fi 70, Sn 80).

(a) Analysis of Variance of Log-Log Transformed Data

Table 2 gives the analysis of variance data.

Table 2. Analysis of variance (log-log transformed data).

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Regression	0.30136	1	0.30136
About Regression	0.23608	k-2 = 5	0.04722
Within Group	0.06578	N-k = 4	0.01644
Residual	0.30186	N-2 = 9	0.03354
Total	0.60322	N-1 = 10	

Working hypotheses: HYP(0): $E(y|x) = k + mx$, or regression is linear

(Where $y = \log_{10} H_o(E_p)$ and $x = \log_{10} E_p$)

HYP(1): not linear

Test statistic: $F = \dfrac{\text{About-Regression Mean Square}}{\text{Within-Group Mean Square}}$

$= \dfrac{0.04772}{0.01644}$ (from Table 2)

$= 2.90$

Level of significance: $\alpha = 0.05$

Critical region: $F > F_{1-\alpha,k-2,N-k}$. With $N = 11$, $k = 7$ this becomes:

$$F > F_{0.95,5,4} = 6.26$$

Decision: since $F \not> 6.26$ the data do not support rejection of the hypothesis that the relation between $\log_{10}H_o(E_p)$ and $\log_{10}E_p$ is linear.

(b) Analysis of Variance of Untransformed Data

Table 3 gives the analysis of variance data.

Table 3. Analysis of variance.

Source	Sum of Squares	Degrees of Freedom	Mean Square
Regression	1.2540×10^{-23}	1	1.2540×10^{-23}
About Regression	0.8790×10^{-23}	$k-2 = 5$	0.1756×10^{-23}
Within Group	0.5872×10^{-23}	$N-k = 4$	0.1468×10^{-23}
Residual	1.4652×10^{-23}	$N-2 = 9$	0.1628×10^{-23}
Total	2.7192×10^{-23}	$N-1 = 10$	

Working hypotheses: HYP(0): $E(y|x) = a + bx$ [where $y = H_o(E_p)$ and $x = E_p$]

HYP(1): Not linear

Test statistic: $F = \dfrac{\text{ABOUT REGRESSION MEAN SQUARE}}{\text{WITHIN GROUP MEAN SQUARE}}$

$= \dfrac{0.1756 \times 10^{-23}}{0.1468 \times 10^{-23}}$ (from Table 3)

$= 1.20$

Level of significance: $\alpha = 0.05$

Critical region: $F > F_{0.95,\ 5,4} = 6.26$

Decision: Since $F = 1.20 \not> 6.26$ the data do not support the rejection of the hypothesis of linearity between $H_o(E_p)$ and E_p.

(c) Summary

A Fisher's F-Test of both the untransformed and the log-log transformed data show that both data sets are consistent with the assumption of linearity.

3.2 REGRESSION ANALYSIS

Regression analysis of the log-log transformed data gives:

$$H_o(E_p) = 3.07 \times 10^{-13} E_p^{0.769} \tag{3}$$

with estimated value of the slope, $\hat{m} = 0.769$ and the estimated variance of $\hat{m}$, $S_{\hat{m}} = \pm 0.257$. Similarly linear regression of the data gives:

$$H_o(E_p) = 5.22 \times 10^{-13} + (1.37 \times 10^{-13}) E_p \tag{4}$$

The estimated variance on the intercept, $S_{\hat{a}}$, $= \pm 9.86 \times 10^{-13}$.

3.3 STUDENT'S t-TEST

With $\hat{m} = 0.769$ and $S_{\hat{m}} = 0.257$ a t-Test does not reject $m = 1.0$ (P = 20 percent) (Li 82).

With $\hat{a} = 5.22 \times 10^{-13}$ and $S_{\hat{a}} = 9.86 \times 10^{-13}$ a t-Test does not reject $a = 0$ (P = 30 percent) (Li 82).

3.4 LINEAR REGRESSION ANALYSIS FORCED THROUGH THE ORIGIN

Since the data are compatible with the assumption of linearity (Fisher's F-Test), with the assumption of $m = 1$ in log-log transformation (Student's t-Test) and with the assumption of $a = o$ when untransformed (Student's t-Test), it is reasonable to fit the data by a line forced through the origin giving:

$$H_o(E_p) = 1.608 \times 10^{-13}\ E_p \tag{5}$$

with $S_{\hat{b}'} = 0.19 \times 10^{-13}$

3.5 CALCULATION OF CONFIDENCE BANDS

Figures 2 and 3 show the 95 percent confidence bands to the lines calculated by regression analysis. For details of these calculations see Lieu et al. (Li 82).

4. CONCLUSIONS

This system of analysis confirms the intuitive conclusion that the quality of the experimental data needs improvement.

The experimental data may be fitted by straight lines either in linear or log-log transformation. The best value of the coefficient, m, is 0.77 ± 0.26, but linearity is not excluded by a Student's t-test (p = 20 percent). A straight line forced through the origin has a slope $(1.61 \pm 0.19) \times 10^{-13}$ Sv m^2/Gev [$(1.00 \pm 0.12) \times 10^{-3}$ Sv m^2/J]. Thus the analysis is not able to distinguish between the conservative assumption (m = 1), and the "theoretical" predictions of m = 0.75 or 0.5.

Two possibilities arise for improving this situation:

(i) To include existing but unpublished data in an analysis of this type. (The present analysis contains only data that were readily available from a search of the published literature. Several shielding experiments carried out at high-energy accelerator laboratories have yielded data which is of great potential value in improving the quality of the regression analysis reported here).

(ii) New experiments, particularly at energies in 100-500 Gev energy region, would be of great value in studying the variation of the Moyer parameter with proton energy.

A better knowledge of the energy variation of $H_o(E_p)$ will be of increasing importance as proton accelerators or storage-rings in the TeV energy region are constructed.

5. ACKNOWLEDGMENTS

The authors would like to thank their colleagues who have provided the data discussed in this paper. We gratefully acknowledge the advice and encouragement of Prof. Chang Wen-Yu of The Institute of High Energy Physics, Beijing; K. Goebel of CERN, and W.D. Hartsough of the Lawrence Berkeley Laboratory during the preparation of this paper. One of us (S.V.T.) would like to thank Prof. J. Coffey, of the Department of Information Systems and Decision Sciences, California State University, Fresno, for his instruction in Statistical

Methods, to M. Simmons of the Computer Center, Lawrence Berkeley Laboratory for much useful advice and encouragement and to the Computer Center at LBL for a Summer Studentship which made his contribution possible.

This work was supported by the European Centre for Nuclear Research, Geneva; by the Institute of High-Energy Physics, Academia Sinica, Beijing; and by the U.S. Department of Energy, with the University of California, under Contract No. DE-AC03-76SF00098.

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LIST OF FIGURES

Figure 1: Schematic Diagram of Shielding Geometry.

Figure 2: $H_o(E_p)$ as a Function of Primary Proton Energy, E_p. The solid line is that calculated by regression analysis and the dashed lines show the 95 percent confidence limits.

Figure 3: $H_o(E_p)$ as a Function of Primary Proton Energy, E_p. Three lines obtained by regression analysis are shown. The solid line shows the best fit to the data assuming a linearity and zero intercept. Two 95 percent confidence bands are shown-- one calculated from the log-log regression analysis, the other calculated from the linear regression analysis.

Note Added in Proof

Tesch has recently referred to new shielding experiments carried out at 350 Gev (Te 83, Co 82). Tesch, in essence, concludes that the data of Cossairt et al. are in reasonable agreement with the assumption m = 1. A preliminary analysis shows that the new experiments are better fitted by m = 0.79, tending to confirm the best value obtained in the present analysis (Th 83). The application of the present techniques to include the new data will be the subject of further work.

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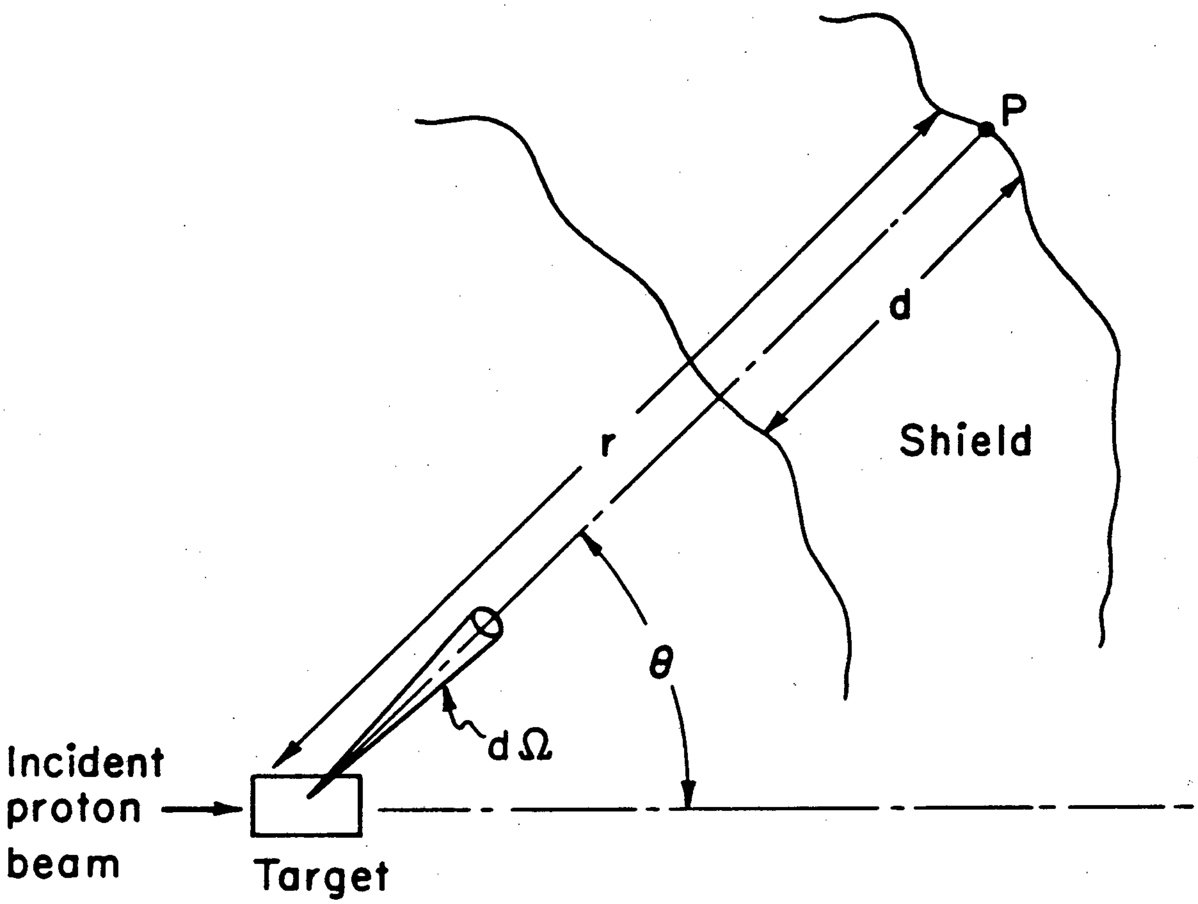


Fig. 1

XBL 733-2478

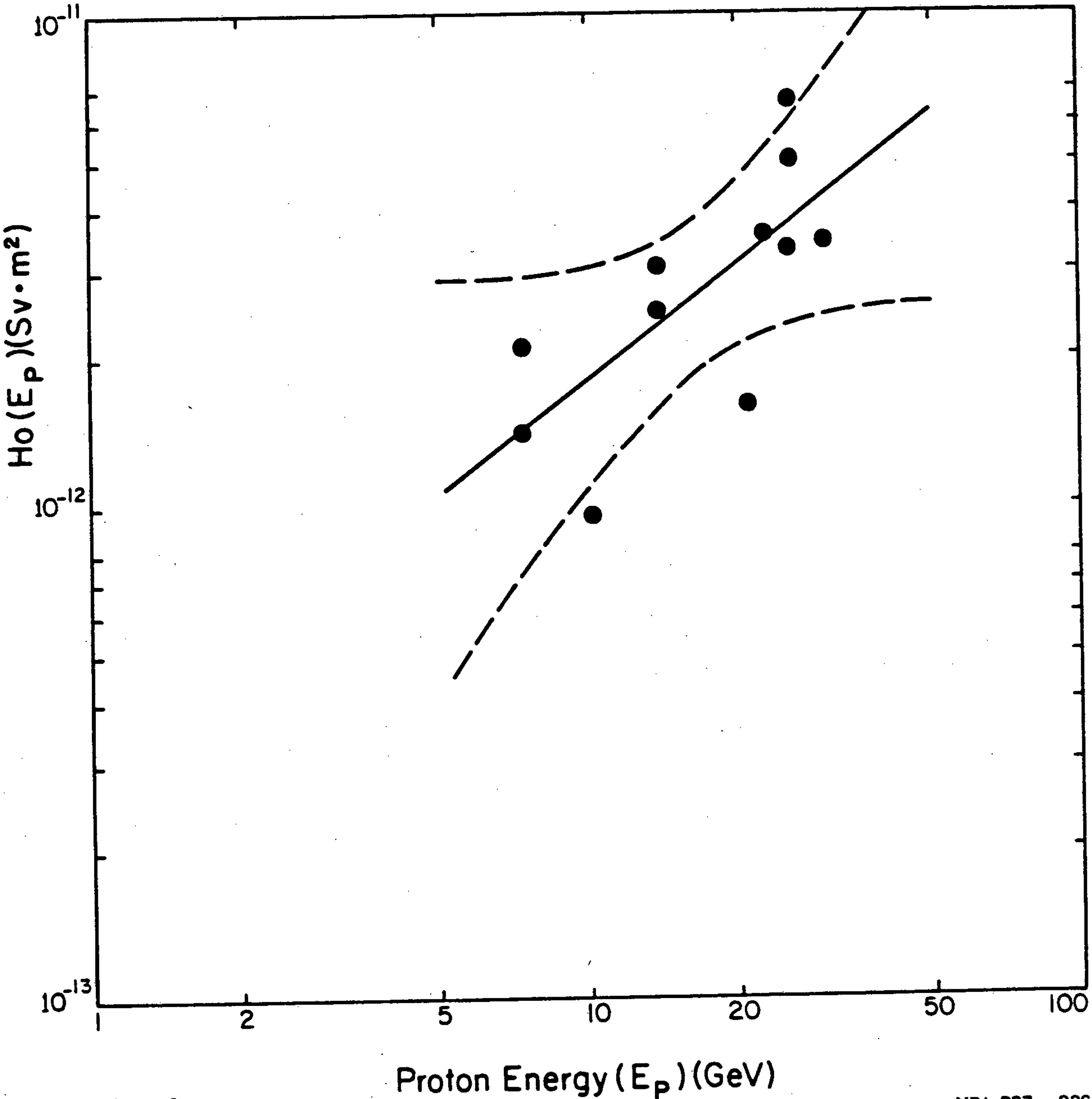
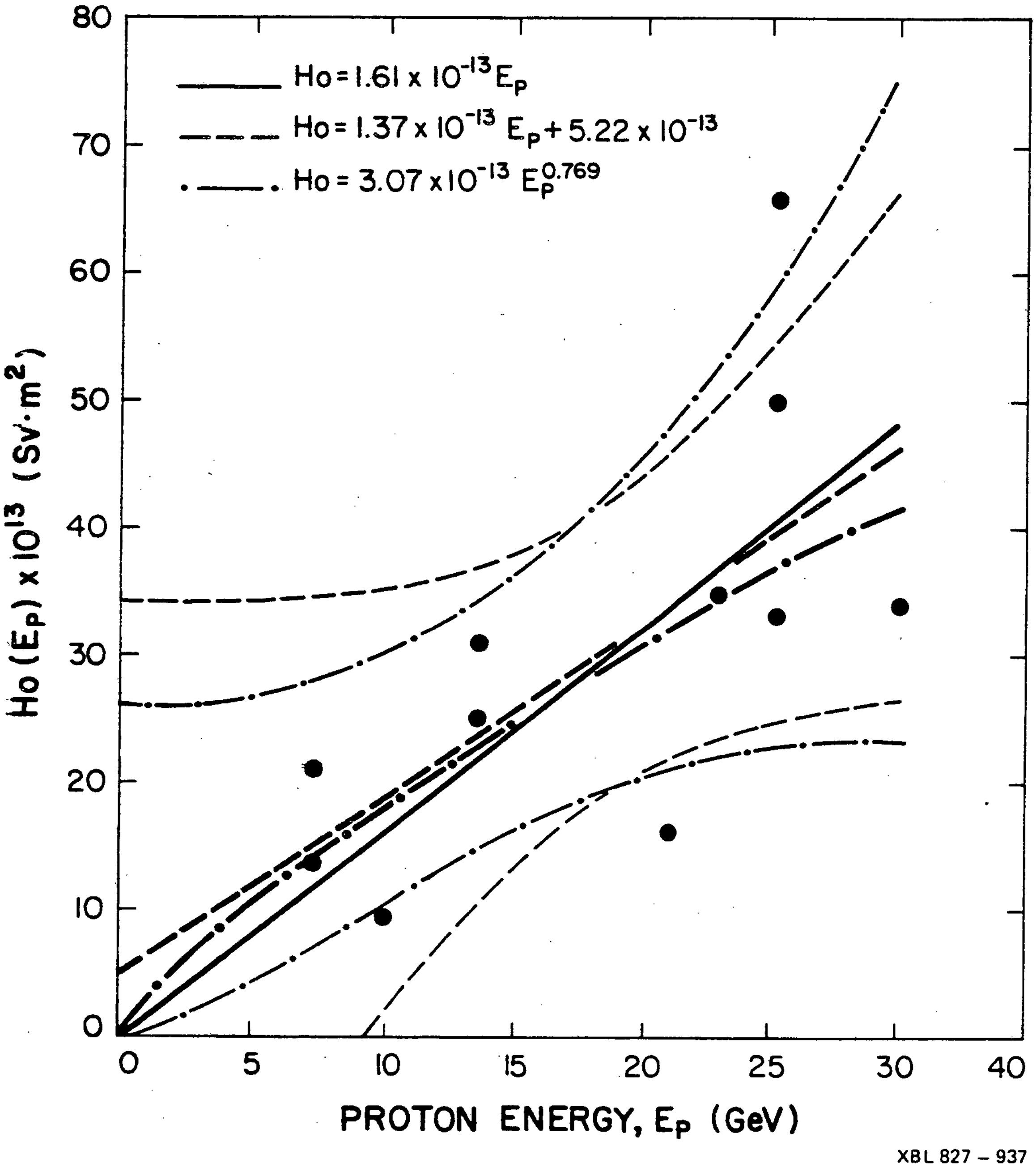


Fig. 2

XBL 827 — 936



80
70
60
50
40
30
20
10
0
Ho = 1.61 x 10^-13 Ep
Ho = 1.37 x 10^-13 Ep + 5.22 x 10^-13
Ho = 3.07 x 10^-13 Ep^0.769
Ho (Ep) x 10^13 (Sv·m^2)
0
5
10
15
20
25
30
40
PROTON ENERGY, Ep (GeV)
XBL 827 – 937

Fig. 3

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